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# Study Guide

Student Edition
Pages 581–586

## *Factoring Differences of Squares*

Use the difference of squares to factor polynomials.

| Difference of Squares | $a^2 - b^2 = (a - b)(a + b) = (a + b)(a - b)$ |
|---|---|

***Example 1:*** Factor $4y^2 - 81z^2$.

$4y^2 - 81z^2 = (2y)^2 - (9z)^2$ **$2y \cdot 2y = 4y^2$ and $9z \cdot 9z = 81z^2$**

$= (2y - 9z)(2y + 9z)$ **Use the difference of squares.**

In some binomials you have to factor a GCF before you can factor the difference of squares.

***Example 2:*** Factor $50a^2 - 72$.

$50a^2 - 72 = 2(25a^2 - 36)$ **The GCF is 2.**

$= 2(5a - 6)(5a + 6)$ **Use the difference of squares.**

***State whether each binomial can be factored as a difference of squares.***

**1.** $a^2 - b^2$ **yes**

**2.** $x^2 + y^2$ **no**

**3.** $a^2 - 36$ **yes**

**4.** $2p - \frac{1}{9}$ **no**

**5.** $\frac{1}{2}m^2 + \frac{1}{4}n^2$ **no**

**6.** $\frac{49}{289}x^2 - 1$ **yes**

**7.** $0.16m^2 + 0.25n^2$ **no**

**8.** $225b^2 - a^2$ **yes**

**9.** $a - 16$ **no**

**10.** $15x^2 + 5$ **no**

**11.** $9y^2 - 4x^2$ **yes**

**12.** $-p^2 + 9q^2$ **yes**

***Factor each polynomial, if possible. If the polynomial cannot be factored, write prime.***

**13.** $m^2 - 16n^2$
**$(m - 4n)(m + 4n)$**

**14.** $4a^2 - 9b^2$
**$(2a + 3b)(2a - 3b)$**

**15.** $x^2 - 64$
**$(x - 8)(x + 8)$**

**16.** $-81 + a^4$
**$(a - 3)(a + 3)(a^2 + 9)$**

**17.** $m^6 - 16n^4$
**$(m^3 - 4n^2)(m^3 + 4n^2)$**

**18.** $-2 + 2y^2$
**$2(y - 1)(y + 1)$**

**19.** $p^2q^2 - \frac{1}{16}$
**$\left(pq - \frac{1}{4}\right)\left(pq + \frac{1}{4}\right)$**

**20.** $\frac{1}{4}z^4 - 25$
**$\left(\frac{1}{2}z^2 - 5\right)\left(\frac{1}{2}z^2 + 5\right)$**

**21.** $\frac{2}{3}x^2 - 9$ **prime**

**22.** $12x^2 - 27y^2$
**$3(2x - 3y)(2x + 3y)$**

**23.** $6 - 54z^2$
**$6(1 + 3z)(1 - 3z)$**

**24.** $(x + y)^2 - w^2$
**$(x + y - w)(x + y + w)$**

**25.** $3x^4 - 75$
**$3(x^2 - 5)(x^2 + 5)$**

**26.** $(n + 7)^2 - 1$
**$(n + 6)(n + 8)$**

**27.** $2p^4 - 32q^4$
**$2(p - 2q)(p + 2q)(p^2 + 4q^2)$**

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## Factoring Trinomials

To factor a trinomial of the form $ax^2 + bx + c$, follow Example 1 below.

***Example 1:*** Factor $2d^2 + 15d + 18$.
The product of 2 and 18 is 36. You need to find two integers whose *product* is 36 and whose *sum* is 15.

| Factors of 36 | Sum of Factors |
|---|---|
| 1, 36 | 1 + 36 = 37 |
| 2, 18 | 2 + 18 = 20 |
| 3, 12 | 3 + 12 = 15 |

$$\begin{aligned} 2d^2 + 15d + 18 &= 2d^2 + (12 + 3)d + 18 \\ &= 2d^2 + 12d + 3d + 18 \\ &= (2d^2 + 12d) + (3d + 18) \\ &= 2d(d + 6) + 3(d + 6) && \textbf{Factor the GCF from each group.} \\ &= (2d + 3)(d + 6) && \textbf{Use the distributive property.} \end{aligned}$$

To factor a trinomial of the form given above when $a = 1$, you need to find only the factors of $c$ whose sum is $b$.

***Example 2:*** Factor $x^2 + 7x + 10$.

Since 2 and 5 are factors of 10 whose sum is 7,
$x^2 + 7x + 10 = (x + 2)(x + 5)$.

The same pattern can be used to factor a trinomial $ax^2 + bx + c$ when $a = 1$ and $c$ is negative. When this occurs, the factors of the trinomial are a *difference* and a *sum*.

***Complete.***

**1.** $x^2 - 5x - 14 = (x + \underline{2})(x - 7)$

**2.** $a^2 + 13a + 36 = (a + 9)(a \underline{+} 4)$

**3.** $p^2 - 25 = (p + 5)(p \underline{-} \underline{5})$

**4.** $x^2 - 6xy - 16y^2 = (x \underline{-} \underline{8y})(x + 2y)$

**5.** $49 - n^2 = (7 \underline{+} \underline{n})(\underline{7} \underline{-} n)$

**6.** $a^4 + 3xa^2 - 10x^2 = (a^2 - \underline{2x})(\underline{a^2} \underline{+} 5x)$

***Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write <u>prime</u>.***

**7.** $x^2 + 12y + 32$
**(y + 4)(y + 8)**

**8.** $x^2 - x - 6$
**(x − 3)(x + 2)**

**9.** $x^2 - 4x - 21$
**(x − 7)(x + 3)**

**10.** $y^2 + 22y + 121$
**(y + 11)(y + 11)**

**11.** $9 - 7n + n^2$
**prime**

**12.** $a^2 - 16a + 64$
**(a − 8)(a − 8)**

**13.** $3x^2 + 2x - 8$
**(3x − 4)(x + 2)**

**14.** $18h^2 - 27h - 5$
**(3h − 5)(6h + 1)**

**15.** $28x^2 + 60x - 25$
**(2x + 5)(14x − 5)**

**16.** $48x^2 + 22x - 15$
**(6x + 5)(8x − 3)**

**17.** $-4y^2 + 19y - 21$
**(4y − 7)(3 − y)**

**18.** $6a^2 - 7a + 18$
**prime**

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## *Factoring Using the Distributive Property*

The distributive property has been used to multiply a polynomial by a monomial. It can also be used to express a polynomial in factored form. Compare the two columns in the table below.

| Multiplying | Factoring |
|---|---|
| $3(a+b) = 3a + 3b$ | $3a + 3b = 3(a+b)$ |
| $x(y-z) = xy - xz$ | $xy - xz = x(y-z)$ |
| $6y(2x+1) = 6y(2x) + 6y(1)$<br>$= 12xy + 6y$ | $12xy + 6y = \mathbf{2} \cdot 2 \cdot \mathbf{3} \cdot x \cdot y + \mathbf{2} \cdot \mathbf{3} \cdot y$<br>$= 6y(2x) + 6y(1)$<br>$= 6y(2x+1)$ |

***Complete.***

1. $9a + 18b = 9($ a $+ 2b)$

2. $12mn + 80m^2 = 4m(3n +$ 20m $)$

3. $7c^3 - 7c^4 = 7c^3($ 1 $- c)$

4. $4xy^3 + 16x^2y^2 =$ $4xy^2$ $(y + 4x)$

***Factor each polynomial.***

5. $24x + 48y$
$24(x + 2y)$

6. $30mn^2 + m^2n - 6n$
$n(30mn + m^2 - 6)$

7. $q^4 - 18q^3 + 22q$
$q(q^3 - 18q^2 + 22)$

8. $a + 8a^2b - ab$
$a(1 + 8ab - b)$

9. $55p^2 - 11p^7 + 44p^5$
$11p^2(5 - p^5 + 4p^3)$

10. $14c^3 - 42c^5 - 49c^4$
$7c^3(2 - 6c^2 - 7c)$

11. $4m + 6n - 8mn$
$2(2m + 3n - 4mn)$

12. $14y^3 - 28y^2 + y$
$y(14y^2 - 28y + 1)$

13. $48w^2z + 18wz^2 - 36wz$
$6wz(8w + 3z - 6)$

14. $9x^2 - 3x$
$3x(3x - 1)$

15. $96ab + 12a^2b - 84ab^3$
$12ab(8 + a - 7b^2)$

16. $45s^3 - 15s^2$
$15s^2(3s - 1)$

17. $18b^2a - 4ba + 7ab^2$
$ab(25b - 4)$

18. $12p^3q^2 - 18p^2q^2 + 30p$
$6p(2p^2q^2 - 3pq^2 + 5)$

19. $-x^5 - 4x^4 + 23x^3 - x$
$x(-x^4 - 4x^3 + 23x^2 - 1)$

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## *Dividing by Monomials*

Study the following exponent rules.

| | Rule | Example |
|---|---|---|
| **Quotient of Powers** | For all integers $m$ and $n$, and any nonzero number $a$, $\frac{a^m}{a^n} = a^{m-n}$. | $q^9 \div q^4 = q^{9-4}$ $= q^5$ |
| **Zero Exponent** | For any nonzero number $a$, $a^0 = 1$. | $4^0 = 1$ $6^0 = 1$ |
| **Negative Exponents** | For any nonzero number $a$ and any integer $n$, $a^{-n} = \frac{1}{a^n}$. | $\frac{r^3}{r^6} = \frac{r \cdot r \cdot r}{r \cdot r \cdot r \cdot r \cdot r \cdot r}$ $= \frac{1}{r^3}$ or $\frac{r^3}{r^6} = r^{3-6} = r^{-3}$ |

***Simplify. Assume no denominator is equal to zero.***

1. $\frac{a^2}{a}$ $a$

2. $\frac{x^5y^3}{x^5y^2}$ $y$

3. $\frac{15a^3}{45a^2}$ $\frac{a}{3}$

4. $\frac{s^{-3}t^{-5}}{(s^2t^3)^{-1}}$ $\frac{1}{st^2}$

5. $\frac{a^5b^3}{a^2b^2}$ $a^3b$

6. $\frac{k^0}{k^7}$ $\frac{1}{k^7}$

7. $\frac{(6a^{-1}b)^2}{(b^2)^4}$ $\frac{36}{a^2b^6}$

8. $\frac{66w^3x^6y^9}{-22wxy^7}$ $-3w^2x^5y^2$

9. $\left(\frac{4m^2n^2}{8m^{-1}l}\right)^0$ $1$

10. $\frac{15x^3}{5x^0}$ $3x^3$

11. $\frac{x^2}{x^3}$ $\frac{1}{x}$

12. $\frac{(3st)^2u^{-4}}{s^{-1}t^2u^7}$ $\frac{9s^3}{u^{11}}$

13. $\frac{b^5}{b^6}$ $\frac{1}{b}$

14. $\frac{x^9}{x^2}$ $x^7$

15. $\frac{24w^7t^4}{6w^3t^2}$ $4w^4t^2$

16. $\frac{9x^2z^5}{-3xz^3}$ $-3xz^2$

17. $\frac{(-x^{-1}y)^0}{4w^{-1}y^2}$ $\frac{w}{4y^2}$

18. $\frac{wt^3x}{wx}$ $t^3$

19. $\frac{w^2}{w}$ $w$

20. $\frac{(a^2b^3)^2}{(ab)^{-2}}$ $a^6b^8$

# Study Guide

## *Multiplying Monomials*

When you multiply monomials, you use the following rules for all numbers $a$ and $b$ and any integers $m$, $n$, and $p$.

| | Rule | Example |
|---|---|---|
| **Product of Powers** | For any number $a$, and all integers $m$ and $n$, $a^m \cdot a^n = a^{m+n}$. | $a^2 \cdot a^6 = a^{2+6}$ $= a^8$ |
| **Power of a Power** | For any number $a$, and all integers $m$ and $n$, $(a^m)^n = a^{mn}$. | $(x^2)^6 = x^{2 \cdot 6}$ $= x^{12}$ |
| **Power of a Product** | For all numbers $a$ and $b$, and any integer $m$, $(ab)^m = a^m b^m$. | $(pq)^4 = p^4q^4$ |
| **Power of a Monomial** | For all numbers $a$ and $b$, and all integers $m$, $n$, and $p$, $(a^m b^n)^p = a^{mp} b^{np}$. | $(s^4t)^3 = (s^4t^1)^3$ $= s^{4 \cdot 3}t^{1 \cdot 3}$ $= s^{12}t^3$ |

***Simplify.***

1. $[n^5(n^2)]$ $n^7$

2. $b(b^4)$ $b^5$

3. $(-7x^2)(x^4)$ $-7x^6$

4. $(2a^2)(8a)$ $16a^3$

5. $(rs)(rs^3)(s^2)$ $r^2s^6$

6. $(x^2y)(4xy^3)$ $4x^3y^4$

7. $\frac{1}{3}(2a^3b)(6b^3)$ $4a^3b^4$

8. $(-5nx)(4x^2)(n^4)$ $-20n^5x^3$

9. $(n^3)^5$ $n^{15}$

10. $(a^4)^6$ $a^{24}$

11. $-3(ab^4)^3$ $-3a^3b^{12}$

12. $(-3ab^4)^3$ $-27a^3b^{12}$

13. $(4x^2b)^3$ $64x^6b^3$

14. $(4x)^2(b^3)$ $16x^2b^3$

15. $(-2m^5n^6)^2$ $4m^{10}n^{12}$

16. $-2m^5(n^6)^2$ $-2m^5n^{12}$

17. $2(3x)^3$ $54x^3$

18. $-3(2x)^5$ $-96x^5$

19. $(-2n^6y^5)(-6n^3y^2)(ny)^3$ $12n^{12}y^{10}$

20. $(-3a^3n^4)(-3a^3n)^4$ $-243a^{15}n^8$

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### *Elimination Using Multiplication*

Some systems of equations cannot be solved simply by adding or subtracting the equations. One or both equations must first be multiplied by a number before the system can be solved by elimination. Consider the following example.

***Example:*** Use elimination to solve the system of equations $x + 10y = 3$ and $4x + 5y = 5$.

$x + 10y = 3$
$4x + 5y = 5$

Multiply $x + 10y = 3$ by $-4$. Then add the equations.

$$\begin{array}{r} -4x - 40y = -12 \\ \underline{4x + 5y = 5} \\ -35y = -7 \\ y = \frac{1}{5} \end{array}$$

Substitute $\frac{1}{5}$ for $y$ into either original equation and solve for $x$.

$$\begin{aligned} x + 10\left(\tfrac{1}{5}\right) &= 3 \\ x + 2 &= 3 \\ x &= 1 \end{aligned}$$

The solution of the system is $\left(1, \frac{1}{5}\right)$.

***Use elimination to solve each system of equations.***

**1.** $3x + 2y = 0$
$x - 5y = 17$
**(2, −3)**

**2.** $2x + 3y = 6$
$x + 2y = 5$
**(−3, 4)**

**3.** $3x - y = 2$
$x + 2y = 3$
**(1, 1)**

**4.** $4x + 5y = 6$
$6x - 7y = -20$
**(−1, 2)**

***Use a system of equations and elimination to solve each problem.***

**5.** The length of Sally's garden is 4 meters greater than 3 times the width. The perimeter of her garden is 72 meters. What are the dimensions of Sally's garden?
**28 meters by 8 meters**

**6.** Anita is $4\frac{1}{2}$ years older than Basilio. Three times Anita's age added to six times Basilio's age is 36. How old are Anita and Basilio?
**Basilio is $2\frac{1}{2}$ yr and Anita is 7 yr.**

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## *Elimination Using Addition and Subtraction*

In systems of equations where the coefficient of the $x$ or $y$ terms are additive inverses, solve the system by adding the equations. Because one of the variables is eliminated, this method is called **elimination.**

***Example:*** Use elimination to solve the system of equations $x - 3y = 7$ and $3x + 3y = 9$.

Add the two equations.

$$\begin{array}{r} x - 3y = 7 \\ \underline{3x + 3y = 9} \\ 4x = 16 \\ x = 4 \end{array}$$

Substitute 4 for $x$ in either original equation and solve for $y$.

$$\begin{aligned} 4 - 3y &= 7 \\ -3y &= 7 - 4 \\ -3y &= 3 \\ y &= -1 \end{aligned}$$

The solution of the system is $(4, -1)$.

***Use elimination to solve each system of equations.***

**1.** $2x + 2y = -2$
$3x - 2y = 12$
$(2, -3)$

**2.** $4x - 2y = -1$
$-4x + 4y = -2$
$\left(-1, -\frac{3}{2}\right)$

**3.** $x - y = 2$
$x + y = -3$
$\left(-\frac{1}{2}, -\frac{5}{2}\right)$

**4.** $6x + 5y = 4$
$6x - 7y = -20$
$(-1, 2)$

**5.** $2x - 3y = 12$
$4x + 3y = 24$
$(6, 0)$

**6.** $0.1x + 0.3y = 0.9$
$0.1x = 0.3y + 0.2$
$\left(\frac{11}{2}, \frac{7}{6}\right)$

***Use a system of equations and elimination to solve each problem.***

**7.** Two angles are supplementary. The measure of one angle is 10 more than three times the other. Find the measure of each angle.
42.5°, 137.5°

**8.** Rema is older than Ken. The difference of their ages is 12 and the sum of their ages is 50. Find the age of each.
Rema is 31 and Ken is 19

**9.** The sum of two numbers is 70 and their difference is 24. Find the two numbers. 23 and 47

**10.** The sum of the digits of a two-digit number is 12. The difference of the digits is 2. Find the number if the units digit is larger than the tens digit. 57

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## *Substitution*

One method of solving systems of equations is by algebraic **substitution.**

***Example:*** Solve $x + 3y = 7$ and $2x - 4y = 6$.

Solve the first equation for $x$.

$$x + 3y = 7$$
$$x = 7 - 3y$$

Substitute $7 - 3y$ for $x$ in the second equation. Solve for $y$.

$$2(7 - 3y) - 4y = -6$$
$$14 - 6y - 4y = -6$$
$$-10y = -20$$
$$y = 2$$

Substitute 2 for $y$ in either one of the two original equations to find the value of $x$.

$$x + 3(2) = 7$$
$$x + 6 = 7$$
$$x = 1$$

The solution of this system is (1, 2).

***Use substitution to solve each system of equations. If the system does not have exactly one solution, state whether it has no solution or infinitely many solutions.***

**1.** $x = 3$
$2y + x = 3$
**(3, 0)**

**2.** $y = 2$
$2x - 4y = 1$
$\left(\frac{9}{2}, 2\right)$

**3.** $y = 3x - 7$
$3x - y = 7$
**infinitely many**

**4.** $y = -x + 3$
$2y + 2x = 4$
**no solution**

**5.** $x + y = 16$
$2y = -2x + 2$
**no solution**

**6.** $x = 2y$
$0.25x + 0.5y = 10$
**(20, 10)**

***Use a system of equations and substitution to solve each problem.***

**7.** How much of a 10% saline solution should be mixed with a 20% saline solution to obtain 1000 milliliters of a 12% saline solution?
**800 mL of 10% solution and 200 mL of 20% solution**

**8.** The tens digit of a two-digit number is 3 greater than the units digit. Eight times the sum of the digits is 1 less than the number. Find the number.
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## *Solving Multi-Step Equations*

When solving some equations you must perform more than one operation on both sides. First, determine what operations have been done to the variable. Then undo these operations in the reverse order.

***Example 1:*** How would you solve $\frac{n}{3} - 7 = 28$?

$\frac{n}{3} - 7 = 28$ First, $n$ was divided by 3. Then 7 was subtracted. } To solve, first add 7 to each side. Then multiply each side by 3.

| Procedure for Solving a Two-Step Equation | 1. Undo any indicated additions or subtractions.<br>2. Undo any indicated multiplications or divisions involving the variable. |
|---|---|

***Example 2:*** $5x + 3 = 23$ **Addition of 3 is indicated.**

$5x + 3 - 3 = 23 - 3$ **Therefore, subtract 3 from each side.**

$5x = 20$ **Multiplication by 5 is also indicated.**

$\frac{5x}{5} = \frac{20}{5}$ **Therefore, divide each side by 5.**

$x = 4$

***Check:***

$5x + 3 = 23$

$5(4) + 3 \stackrel{?}{=} 23$

$20 + 3 \stackrel{?}{=} 23$

$23 = 23$ ✔

***Solve each equation. Then check your solution.***

**1.** $5z + 16 = 51$ **7**

**2.** $14n - 8 = 34$ **3**

**1.** $0.6x - 1.5 = 1.8$ **5.5**

**4.** $\frac{4b + 8}{-2} = 10$ **−7**

**5.** $16 = \frac{d - 12}{14}$ **236**

**6.** $8 + \frac{3n}{12} = 13$ **20**

**7.** $\frac{7}{8}p - 4 = 10$ **16**

**8.** $\frac{g}{-5} + 3 = -13$ **80**

**9.** $-4 = \frac{7x - (-1)}{-8}$ $\mathbf{\frac{31}{7}}$**, or** $\mathbf{4\frac{3}{7}}$

***Define a variable, write an equation, and solve each problem. Then check your solution.***

**10.** Find three consecutive integers whose sum is 96.
***n* + (*n* + 1) + (*n* + 2) = 96; 31, 32, 33**

**11.** Find two consecutive odd integers whose sum is 176.
***n* + (*n* + 2) = 176; 87, 89**